A Method Based on the Principle of Critical Energy for Calculating Flange Joints

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An analysis has been made of the behavior of flange joint components under initial stress loading (pre-clamping) and in service. There have been highlighted the drawbacks in calculating flange joints to be found in currently acknowledged methods that do not take into account: - the influence of the bending moment produced by the total hydrostatic end force on bolt strength and flange rotation; - the presence of cracks in the welding or in its thermal influence zone; - the residual stresses; – the relaxation of the sealing gasket over time. The calculation method proposed in the paper takes into account all these particularities of loading. Furthermore, alongside the strength calculation of bolts and flanges, a proposal has been put forth for the calculation of the flange joint leak-tightness.

Keywords: flanges, bolts, tightness, strength calculation, principle of critical energy

Pressure vessels and industrial pipelines are provided with flange joints. Figure 1 shows a flange joint of the kind usually found in some pressure vessels.

The sealing of this removable assembly is achieved by bolting (6) together two flanges, 1 and 2, with a gasket (3) between them to provide a seal. The sealing gasket is compressed as much as necessary in order to ensure the leak-tightness of the flange joint while in service when the vessel is undergoing an internal pressure, p.

The internal pressure causes the total hydrostatic end force $F_H$, which causes the flanges to rotate and deforms the shells to which they are joined. The flanges rotation has two effects (Fig. 2):
- applies further stress upon bolts in bending;
- determines the variation of the sealing gasket thickness and reduces the effective sealing gasket width.

The bolt undergoes a tensile stress caused by the clamping force applied by fitting in bolting in nut.

On top of this, while in service, one should add the bending moment produced by the total hydrostatic end force, $F_H$ (Fig. 2).

The flanges undergo a bending stress caused by bolt force which tends to rotate them around the circumference of the circle diameter $D_3$ (characteristic of the gasket) and, separately, by the total hydrostatic end force, $F_H$, which rotates them around the circumference passing through the axes of the bolts ($D_2$). The total rotation is $2\theta$ (Fig. 2).

The following issues arise:
-ensuring the mechanical strength of the bolts under traction load in service with force $F_{b,p}$;
-ensuring flange rigidity and end stiffness of each shell, such that flange rotation, $2\theta$, and flange deformation between

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bolts might be sufficiently small so that the thickness variation of the gasket in service should be insignificant and should not affect the tightness;
- the deformation of the sealing gasket produced by the clamping force should ensure leak-tightness throughout the prescribed period, also taking into account its relaxation over time.

Current calculation methods [1-3] do not take into account:
- the presence of cracks in the welding or in its thermal influence zone;
- residual stresses in flange joint components;
- the relaxation of the sealing gasket over time, which affects its sealing capability;
- the influence of the mean stress on flange joint life in the case of variable stress loading (fatigue).

Further down there is proposed a method of calculating flange joints in order to eliminate the reported drawbacks. With this end in view, the principle of critical energy has been used [4-7].

**Stress loading of flange joint components**

a. The force of the bolts in initial clamping at ambient pressure (pre-clamping) is calculated by the relationship (Figure 1),

\[ F_{b,0} = \pi \cdot D_1 \cdot b \cdot q_0, \]

where \( b \) is the effective or computation width of the sealing gasket, which depends on its reference width, \( b_0 \) [1-0]; \( q_0 \) is the gasket pressure on the initial clamping recommended by the gasket manufacturer, or taken from the calculation norm.

b. The force of the bolts in service, under the vessel pressure, \( p \), if the temperature difference between the flanges \( (T_f) \) and the bolts \( (T_b) \) is null, \( \Delta T_{f,b} = T_f - T_b = 0 \), has the expression [1-3],

\[ F_{b,e} = F_H + F_G, \]

where,

\[ F_H = \frac{\pi \cdot D_3^2}{4} \cdot p_c - \text{is the total hydrostatic end force;} \]

\[ F_G = 2 \cdot \pi \cdot D_1 \cdot b \cdot p_n - \text{is the compression load on gasket the ensure tight joint;} \]

\( p_c \) – the calculation pressure;

\[ p_{sp} = m \cdot p_c - \text{the sealing pressure, where } m>1 \text{ depends on the type and particularities of the sealing gasket.} \]

From the equality \( F_{b,0} = F_{b,e} \), one gets the pressure on the gasket needed for pre-clamping \( (t=0) \),

\[ q_{0,n} = \frac{F_{b,0}(t=0)}{\pi \cdot D_3 \cdot b}. \]

If,

\[ q_0 \geq q_{0,n} - \text{the initial clamping ensures tightness in service;} \]

\[ q_0 < q_{0,n} - \text{the initial clamping is insufficient and in service the leak tightness may be affected.} \]

The total hydrostatic end force, \( F_H \), consists of the unclamping force determined by the action of the inner pressure on the cross-sectional area of the vessel, \( F_D \), and on the flange, between diameters \( D_3 \) and \( D_1 \), written as \( F_T \),

\[ F_H = F_D + F_T, \]

where (Fig. 1,b),

\[ F_D = \frac{\pi \cdot D^2}{4} \cdot p_c \]

\[ F_T = \frac{\pi}{4} (D_1^2 - D^2) \cdot p_c \]

Consequently, in service,

\[ F_{b,s} = F_D + F_T + F_G. \]

The bending moment applied to the bolts, determined by the partial unclamping forces \( F_D \) and \( F_T \) has the expression (Fig 1, b),

\[ M_{b,H} = a_D \cdot F_D + a_T \cdot F_T, \]

where, for the case represented in Figure 1,

\[ a_D = \frac{D_1 - D}{2}; \]

\[ a_T = \frac{1}{2} \left( \frac{D_1 - D}{2} + \frac{D_3 - D}{2} \right) - \frac{1}{4} \left( D_3 - D \right). \]

c. Bolt stress loading in case \( \Delta T_{f,b} \neq 0 \).
If flange temperature (T) is different from bolt temperature (Tb), the temperature difference $\Delta T_{fb} = T - Tb$ generates an additional force $F(\Delta T)$ added to the $F_G$ component. Consequently, in relations (2) $F_G$ is replaced by

$$F_G' = F_G + F(\Delta T),$$

where $F(\Delta T)$ is calculated with the relation given in works [8-10].

The total force in the bolts in service, becomes,

$$F_{b,s}^{*} = F_{b,s} + F(\Delta T).$$

**d. Flange stress loading.** First ($F_{b,0}$), the force in the bolts is applied during pre-clamping — the internal pressure is introduced, the stress loading caused by it emerges, which generates the bending moment ($M_{b,0}$).

When clamping the bolts, force $F_{b,0} \geq F_{b,s}$ causes the flanges to rotate around the circumference of diameter $D_3$, characteristic of the sealing gasket location. A bending moment is created,

$$M_{b,0} = a_g \cdot F_{b,0} = a_g \cdot F_{b,s'},$$

where for the case in Figure 1,

$$a_g = \frac{D_2 - D_1}{2}.$$

When in service (under pressure), each flange is subjected to a total bending moment, when $\Delta T_{fb} = 0$,

$$M_{b,f} = M_{b,H} + M_{b,b} = a_g \cdot F_D + a_g \cdot F_T + a_g \cdot F_{b,0}.$$

Optimally, when $F_{b,0} = F_{b,s} = F_H + F_G$, one acquires the total bending moment applied to the flanges,

$$M_{b,s} = a_g \cdot F_D + a_g \cdot F_T + a_g \cdot (F_D + F_T + F_G) \quad \text{or} \quad M_{b,s} = (a_g \cdot a_0) \cdot F_D + (a_g \cdot a_T) \cdot F_T + (a_g \cdot F_{b,s}),$$

If $\Delta T_{fb} \neq 0$, in relations (13), one replaces $F_G$ with $F_G'$ and one gets the total bending moment,

$$M_{b,s} = (a_g + a_\sigma) \cdot F_D + (a_T + a_\sigma) \cdot F_T + a_g \cdot F_{b,s}'.$$

As $F_G' \neq F_G$, it is also obvious that $M_{b,s} \neq M_{b,f}$.

e. **Stresses in bolts and flanges**

- Bolts undergo tensile stresses featuring force $F_{b,0}$, $F_{b,s}$ or $F_{b,s}'$ and bending stress $M_{b,H}$.

Stresses in bolts have the following expressions:

$$\sigma_b = \frac{F_{b,s}}{A_{b,\min}} - \text{tensile stress in the area featuring minimum bolt cross-sectional area (} A_{b,\min});$$

$$\sigma_{b,b} = \frac{M_{b,H}}{W_b} - \text{bending stress in the bolt,}$$

where $A_{b,\min} = \frac{\pi}{4} \cdot d_{b,\min}^2$ and $W_b = \frac{\pi \cdot d_{b,\min}^3}{16}$ while $d_{b,\min}$ - the minimum diameter of the bolt.

The maximum bending stress in the flange is,

$$\sigma_{f,b} = \frac{M_{b,s}}{W_f},$$

where $M_{b,s}$ is given by the relation (13) or (14), and the modulus of strength of the flange section is, $W_f = \frac{\pi \cdot D \cdot h^2}{6}$.

The **maximum flange bending stress** can be calculated with some relationships to be found in works [10-13]. For example, if:

- the width of the flange plate is relatively small compared to the median diameter of the flange, the corresponding stress can be calculated with the relationship,

$$\sigma_{f,s,max} = \frac{M_{b,T}}{W_f};$$

- the width of the flange plate is not small as compared to the median diameter of the flange plate,

$$\sigma_{f,s,max} = \frac{6 \cdot M_s}{h^2 \cdot R \cdot \ln(D_i/D)}.$$

**f. Stress in the shell to which the flange is welded**

In order to check the shell thickness (4 or 5 in Fig. 1) to which the flange is welded, the bending moment theory [12-14] is used. For example, for the flange in Figure 1, one calculates the stress in the shell wall whose calculation thickness is $\delta < s$, with the relationship,
\[
\sigma_b = \pm \frac{6 \cdot M_0}{\delta^2}, \tag{15}
\]

where \(M_0\) is the bending moment acting in the junction section between the flange and the shell (between 4 and 1 in Figure 1) and which can be calculated with the Eq. [12],

\[
M_b = 0.5 \cdot F_b \cdot (D_b - D) \cdot \frac{1}{1 + \frac{k \cdot h}{2} \left(1 - \frac{\mu}{k \cdot D} \right)} \ln \left(\frac{D_1}{D}\right),
\]

where \(k = \frac{3(1-\mu^2)}{\sqrt{R \cdot \delta}}\) is the damping coefficient and \(\mu\) is Poisson's coefficient for the flange material. For steel, with \(\mu \approx 0.3\), the result is \(k = \frac{1.285}{\sqrt{R \cdot \delta}}\).

**Calculation for testing flange joints**

In design, a flange joint is first selected based on the service parameters. Then its components are verified. At present [1-3], the calculation of the flange joint is only a test of the strength of the bolts and the flange, in the simple case when \(\Delta T_{fb} = 0\), on the basis of data comprising many diagrams whose logic is not generally clear to the designer.

In the present paper, instead of resorting to the acknowledged method of strength calculation [1-3], we proposed a new method of calculation where one takes into account:
- the temperature difference between flange and bolts \(\Delta T_{fb} \neq 0\);
- the time dependence of the variables involved;
- the residual stresses, \(\sigma_{res}\), through residual stress participation \(P_{res}(t)\);
- total deterioration to the assembly components, \(D_T(t)\).

Such an approach to the calculation of strength is possible due to the use of the principle of critical energy [4-7]. In addition to the calculation of strength, our work has separately put forth a leak-tightness calculation as well.

**a. Bolt strength testing.**

Bolt and flange materials are believed to behave linearly-elastic, according to Hooke's law,

\[
\sigma = E \cdot \varepsilon, \tag{16}
\]

where \(\sigma\) is the normal stress, \(\varepsilon\) - strain, \(E\) - the modulus of longitudinal elasticity (Young).

Stress loading must not exceed the admissible state, which means that the maximum stresses are lower than the yield limit and, consequently, the behavior of the material is linear-elastic according to the law (16).

In service, the bolts undergo a tensile force and a bending moment.

This is a case of stress superposition to be solved on the basis of Energonics [4;7], using the principle of critical energy [4-7] and the law of equivalence of processes and phenomena [15] from Energonics.

- The calculation is based on the use of the concept of participation of the specific energy introduced by stress loading, in relation to the admissible state, and the admissible participation concept, both of which are non-dimensional variables dependent on the behavior of the material under stress.

For bolts the following relationships are obtained:
- the total contribution of the specific energies corresponding to the stress loadings, with respect to the allowable status,

\[
P_T^* = \left(\frac{\sigma_b}{\sigma_{b,al}}\right)^2 + \left(\frac{\sigma_{b,b}}{(\sigma_{b,b,al})}\right)^2, \tag{17}
\]

where \(\sigma_b\) is the tensile stress in the bolt produced by force \(F_b\) and \(\sigma_{b,al}\) allowable stress in the bolt under tensile stresses; \(\sigma_{b,b}\) bending stress in the bolt produced by the bending moment \(M_{b,an}\) while \(\left(\sigma_{b,b,al}\right)\) - the allowable stress in the bolt during bending;
- the allowable participation has the expression [7; 16],

\[
P_{T,al} = D_T(t) \cdot P_{res} \cdot P_{al}, \tag{19}
\]

where \(D_T(t)\) is the total deterioration with the respect to the allowable state; \(P_{res}\) - participation of residual stresses with respect to the allowable state.

From a practical point of view, if,

\[
\begin{cases}
P_T^* \leq P_{al} & \text{the stress loading state is admissible;} \\
P_T^* > P_{al} & \text{the stress loading state is not admissible.}
\end{cases}
\]

The expression of the admissibility state can also be made on the basis of the concept of equivalent bending stress in the bolt \(\left(\sigma_{b,b}\right)_{eq}\). The total participation with respect to the allowable state, in this case, has the expression,
Using the law of equivalence of processes and phenomena [4; 7; 15], there results the equivalent force in bending stress,
\[
P'_t = \left( \frac{\sigma_{b,b}}{\sigma_{b,b}^*} \right)^2. \tag{20}
\]

From relations (17) and (20), according to the law of equivalence of processes and phenomena [4; 7; 15], there results the equivalent force in bending stress,
\[
\left( \sigma_{b,b} \right)_{eq} = \left( \sigma_{b,b}^2 + \left( \frac{\sigma_{b,b}^*}{\sigma_{b,b}^*} \right)^2 \cdot s_b^2 \right)^{0.5}. \tag{21}
\]

The condition of stress loading admissibility is, in this case,
\[
\left( \sigma_{b,b} \right)_{eq} \leq \left( \sigma_{b,b}^* \right)_{al}, \tag{22}
\]
where, however, for generality \( \left( \sigma_{b,b}^* \right)_{al} \) is expressed in terms of the damage and residual stresses, according to the relationship established on the basis of the principle of critical energy [16;17],
\[
\left( \sigma_{b,b}^* \right)_{al} = \left( \sigma_{b,b} \right)_{al} \left[ 1 - D_f(t) - P_{res} \right]^{0.5}, \tag{23}
\]
where \( \left( \sigma_{b,b} \right)_{al} \) is the allowable bending strength of the undeteriorated bolt material, without cracks and without residual stresses. The deterioration and the residual stresses in the brackets refer to the bolt.

- **The strength condition for the bolts** can be solved:
  - based on the principle of critical energy, according to the first relation (19);
  - based on the law of equivalence of processes and phenomena, according to relations (22).

  **b. Flange strength testing.** The condition for flange strength is,
  \[
  \sigma_{f,b} \leq \left( \sigma_{f,b}^* \right)_{al}, \tag{24}
  \]
where \( \left( \sigma_{f,b}^* \right)_{al} \) is the bending allowable stress for the flange material, calculated with the relationship (25) established on the basis of the principle of critical energy,
\[
\left( \sigma_{f,b}^* \right)_{al} = \left( \sigma_{f,b} \right)_{al} \left[ 1 - D_f(t) - P_{res} \right]^{0.5}, \tag{25}
\]
where \( \left( \sigma_{f,b} \right)_{al} \) is the allowable strength under bending for the flange material without deterioration, with no cracks and no residual stresses. Deterioration and residual stresses in parentheses refer to flange (fl).

- **c. Leak-tightness condition**
  The leak tightness condition results from the observance of the first relationship (4). It should be remembered that the gaskets made of elastoviscous materials relax over time. Gasket relaxation is equivalent to the reduction in time of factor \( m = m(t) \) in the relationship of sealing pressure \( P_{sp} \). It should be also considered that the sealing material often behaves non-linearly [18; 19].

  As a result, the sealing force, \( F_G \) or \( F_G^* \), decreases over time. This causes the bolt force to decrease over time, which explains the fact that, in some cases, sometimes “inexplicably”, the flange joint loses its tightness.

  At present, design rules [1-3] prescribe unique time independent values for the \( m \) factor of each type of sealing gasket. This problem, regarding the dependence of \( m \) on service time must be investigated and remedied.

  After an operating time \( t \), the effective pressure on the gasket is,
  \[
  q_{ef}(t) = \frac{F_{ef}(t) \cdot F_{sp}(t)}{ \pi \cdot D_h \cdot b}. \tag{26}
  \]
  The leak - tightness condition is verified if:
  \[
  q_{ef}(t) \geq q_0. \tag{27}
  \]

- **d. Strength testing of the shell to which the flange is welded to.**

  The effective stress, \( \sigma_b \), calculated for example with relation (15), is compared to the allowable stress \( \sigma_{b,al} \leq 2 \cdot \sigma_\gamma(T) \) in the joint section, where \( \sigma_\gamma(T) \) is the yield stress of the shell material at the calculation temperature. It is imperative that,
  \[
  \sigma_b \leq \sigma_{b,al}. \tag{28}
  \]

- **e. Fatigue life prediction.** In the case of cyclic loading may be calculate the fatigue life using the results obtained in the paper [20], concerning the simultaneous cyclic loading with blocks of normal and shear stresses.

**Analysis of the proposed calculation method**

Practically, the proposed calculation of flange joints is a calculation meant to test the strength of the bolts, flanges and leak-tightness.

The proposed calculations in this paper are more general than those in official regulations, but they are easier to apply because they rely directly only on primary variables that are easy to understand and process by the designer.

The proposed calculation method comprises the following sequence:
- set the regime parameters \( (p; T_j; T_s) \) and calculate pressure \( (p_i) \);
- choose the flange joint and highlight the characteristic dimensions required for the calculation (Figure 1);
- select the sealing gasket and calculate the sealing pressure ($p_{op}$);
- calculate forces ($F_{b0}$, $F_{b}$, $F_{b0}^t$, ...);
- calculate bending moments ($M_{b0}$, $M_{b}$, $M_{b0}^t$);
- calculate stresses in bolts ($\sigma_{b0}$, $\sigma_{b}$);
- calculate the total participation of the specific energies corresponding to the bolt loads with respect to the allowable state ($P_T^*$);
- calculate the allowable participation for bolts ($P_{al}$);
- test bolt strength ($P_T^* \leq P_{al}$);
- calculate the maximum bending stress in the flange ($\sigma_{f,b}^{max}$);
- calculate the maximum allowable stress in the flange ($\sigma_{f,b}^{al}$);
- test flange strength (rel. (24));
- test the strength of the shell to which the flange is welded (rel. (28));
- calculate the pressure on the gasket required by pre-clamping (rel. (4));
- calculate the effective pressure on the gasket ($q_{eff}(t)$) after operating for a time, $t$, according to relationship (rel. (26));
- test the leak-tightness condition (rel. (27)).

**Conclusions**

New relationships have been proposed for calculating the strength of flange joints corresponding to actual conditions.

When calculating the stress in the bolts: -one also took into consideration the bending moment caused by the unclamping force; - the superposition of stress loads caused by the bolt pressure and bolt force based on the principle of critical energy.

The admissible reference condition for bolts also includes the influences of residual stresses and deterioration.

When calculating flange strength, one took into consideration, in writing the expression for allowable stress, the possible influence of residual stresses as well as the flange deterioration.

One also introduced, distinctly, a calculation for testing the flange joint leak-tightness as well as the strength of the shell to which the flange was welded.

The sequel of calculations presented in the paper allows the implementation of the proposed relations and represents a new complete method for calculating flange joints.

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