Research Concerning the Movement Equation of the Mechanism of the Conventional Sucker Rod Pumping Units

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Establishing and then solving the movement equation of the mechanism of the conventional sucker rod pumping units allows determining the variation on the cinematic cycle of the angular speed of the cranks as a dynamic response to the motor and resistant actions on the component elements. The evaluation as accurate as possible of the dynamic response depends on a more precise determination of the variation on the cinematic cycle of the synthesis parameters that occur in the movement equation of the mechanism of the pumping units. In the paper is established the movement equation of the mechanism of the conventional pumping units and are presented a series of results regarding the variation on the cinematic cycle of some synthesis parameters in the case of the mechanism of a C-640D-305-120 pumping unit. Experimental records have been processed with the Total Well Management program and the simulations have been performed with a computer program developed by the author using the Maple programming environment.

Keywords: sucker rod pumping unit, dynamics, movement equations, synthesis parameters

The analysis of the conventional pumping installations in order to improve their operation requires the completion of complex studies both on the level of the sucker rods pumping column and also concerning the surface equipment, especially the pumping unit mechanism [1-8]. Regarding the mechanism of the pumping units, from the perspective of their dynamics, the study of the movement equation is extremely important for highlighting the dynamic response to the motor and resistant actions on the component elements [9-10].

In the paper is presented the movement equation of the mechanism of the conventional pumping units and are outlined the calculation relations for the determination of the synthesis parameters intervening in this equation. A series of results regarding the variation on the cinematic cycle of some synthesis parameters in the case of the mechanism of a C-640D-305-120 pumping unit are also presented. The experimental records have been processed with the *Total Well Management* program [12], and the simulations were performed with a computer program developed by the author using the Maple programming environment [11].

Experimental part

The experimental records processed with the program *Total Well Management* [12] have been obtained from a well serviced by a C-640D-305-120 pumping unit manufactured by *Lufkin* [13]. For establishing the variation on a cinematic cycle of the reduced moment M_{red}^{g} corresponding to the weight forces of the component elements of the mechanism of the conventional pumping units that appears in the movement equation were used the records concerning the variation of the force at the polished rod and of the motor moment at the crankshaft for the stroke 70 (fig. 1) and the variation of the angular speed of the cranks expressed in rot/min during the stroke 70 (fig. 2).

Verifying the simulation results has been done using the experimental records obtained for the variation of the motor moment at the crankshaft and of the angular speed of the cranks expressed in rot/min during the strokes 71 and 72 (fig. 3÷6).

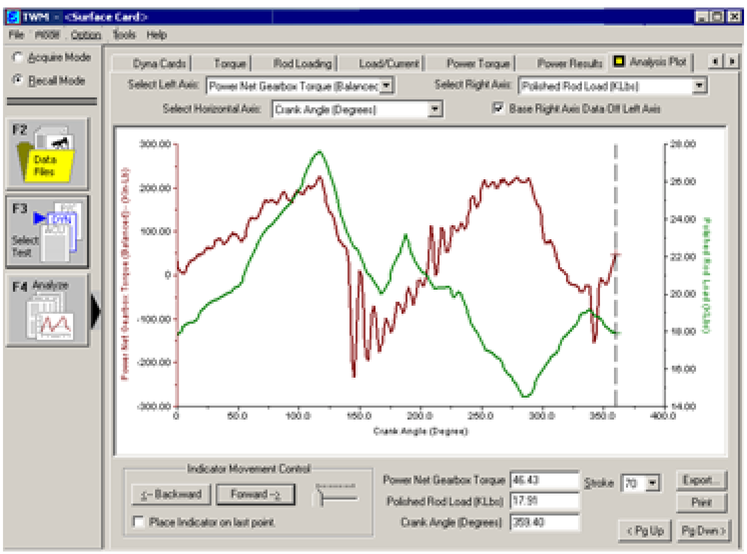

Fig. 1. The variation of the force at the polished rod and of the motor moment at the crankshaft during the stroke 70

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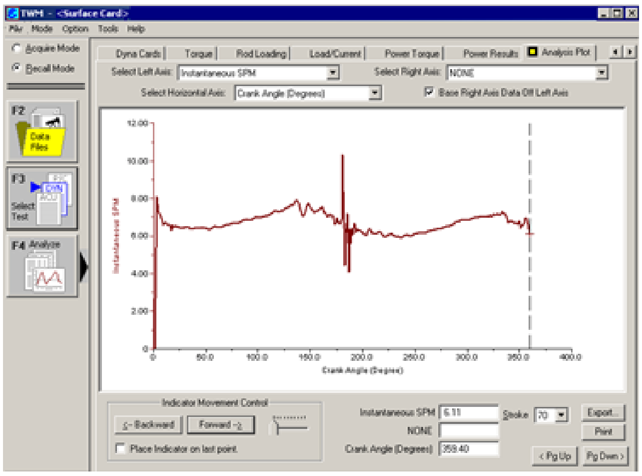

Fig. 2. The variation of the angular speed of the cranks expressed in rot/min during the stroke 70

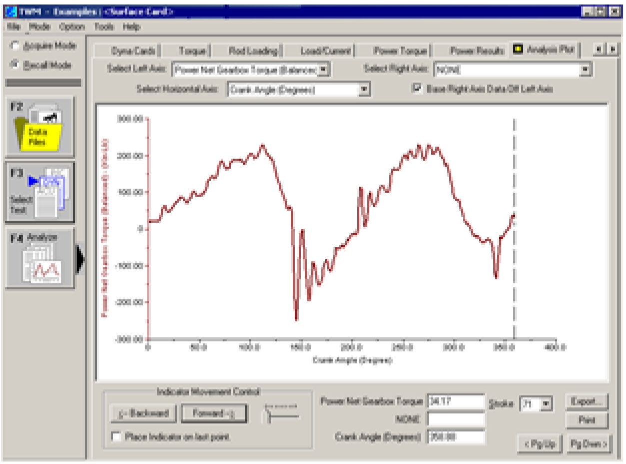

Fig. 3. The variation of the motor moment at the crankshaft during the stroke 71

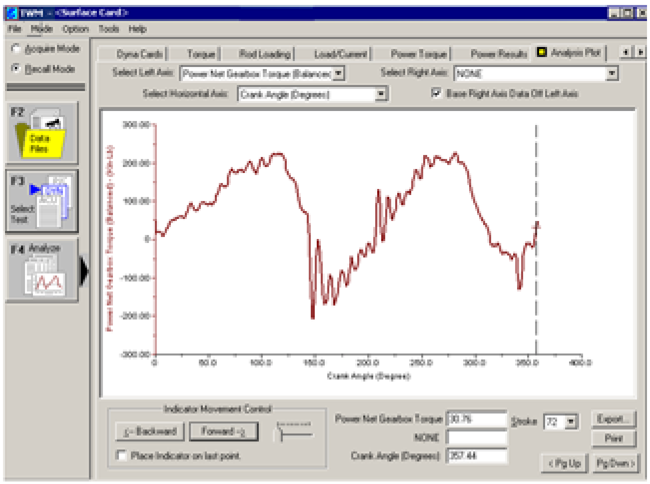

Fig. 4. The variation of the motor moment at the crankshaft during the stroke 72

The movement equation of the mechanism of the conventional pumping units

The movement equation in the case of a plane mechanism can be expressed in the following form [10]:

$$\frac{1}{2}\cdot\frac{dJ_{red}}{d\varphi_1}\cdot\omega_1^2+J_{red}\cdot\omega_1\cdot\frac{d\omega_1}{d\varphi_1}=M_{red} \qquad (1)$$

where: ω_1 is the angular speed expressed in rad/s of the driving crank of the mechanism; φ_1 is the driving crank angle; J_{red} is the reduced mass moment of inertia and M_{red}

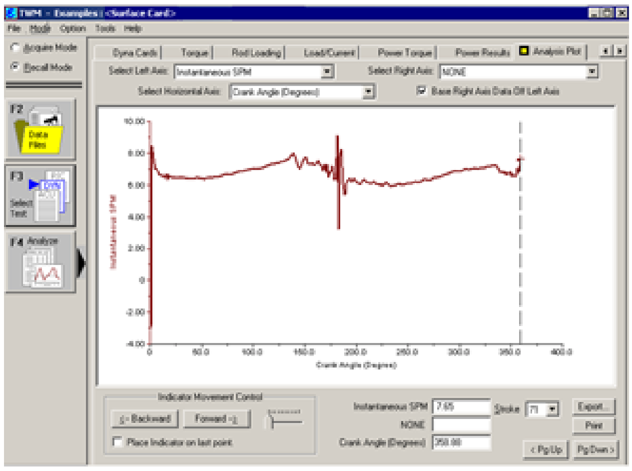

Fig. 5. The variation of the angular speed of the cranks during the stroke 71

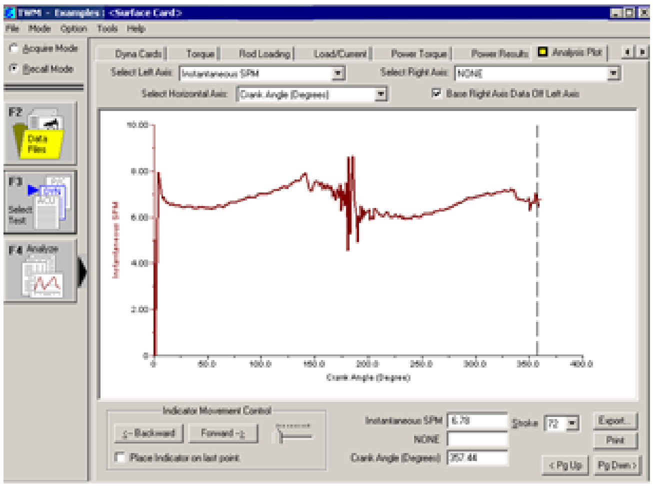

Fig. 6. The variation of the angular speed of the cranks during the stroke 72

is the reduced moment. J_{red} and M_{red} may be calculated with the following relations [10]:

$$J_{red}=\sum_j\left(m_j\cdot\left(\frac{v_{Cj}}{\omega_1}\right)^2+J_{Cj}\cdot\left(\frac{\omega_j}{\omega_1}\right)^2\right) \qquad (2)$$

$$M_{red}=\frac{1}{\omega_1}\cdot\sum_j\left(\overline{F}_j\cdot\overline{v}_{Cj}+\overline{M}_j\cdot\overline{\omega}_j\right) \qquad (3)$$

where: m_j is the mass of the component element *j* of the mechanism; J_{Cj} is the mass moment of inertia of the element *j* calculated relative to its center of mass; $\overline{v}_{Cj}$ is the speed of the mass center of the element *j*; ω_j is the angular speed of the element *j*; F_j is the resultant force acting on the element *j* reduced in its mass center; M_j is the resultant moment of all the forces acting on the element *j* calculated relative to its center of mass.

In figure 7 is presented the mechanism of the conventional pumping units. C_1, C_2 and C_3 are the mass centers of the cranks, connecting rods and of the rocker, respectively; m_{CG} is the total mass of the balancing counterweights; m_{L1} represents the mass of the connecting bearings between the cranks and the connecting rods; m_{L2} is the mass of the spherical connecting bearing between the connecting rods and the rocker; m_{tr} is the mass of the equalizer traverse; m_{CB} is the mass of the rocker head considered to be concentrated in point *D'*; M_m is the motor moment at the crankshaft and F is the force acting at the end of the polished rod.

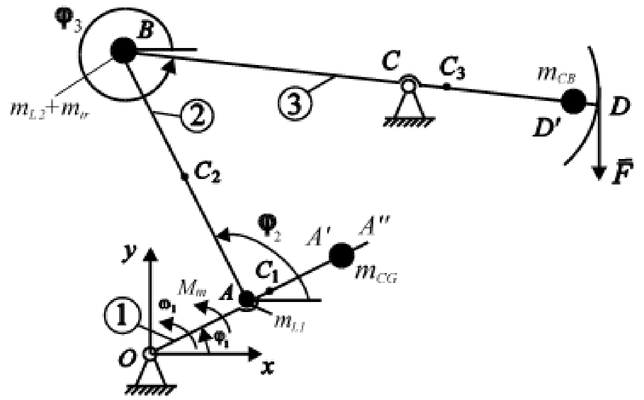

Fig. 7. The mechanism of the conventional pumping units

By applying the relations (2) and (3) in the case of the mechanism of the conventional pumping units, the reduced mass moment of inertia J_{red} and the reduced moment M_{red} may be calculated with the following relations:

$$J_{red}=m_1\cdot\left(\frac{v_{C1}}{\omega_1}\right)^2+m_2\cdot\left(\frac{v_{C2}}{\omega_1}\right)^2+m_3\cdot\left(\frac{v_{C3}}{\omega_1}\right)^2+m_{L1}\cdot\left(\frac{v_A}{\omega_1}\right)^2+(m_{L2}+m_{tr})\cdot\left(\frac{v_B}{\omega_1}\right)^2+ \\ +m_{CB}\cdot\left(\frac{v_{D'}}{\omega_1}\right)^2+m_{CG}\cdot\left(\frac{v_{A'}}{\omega_1}\right)^2+J_{C1}+J_{C2}\cdot\left(\frac{\omega_2}{\omega_1}\right)^2+J_{C3}\cdot\left(\frac{\omega_3}{\omega_1}\right)^2 \qquad (4)$$

$$M_{red}=M^g_{red}+M^f_{red}+M_m \qquad (5)$$

where: m_1, m_2 and m_3 represents the mass of the cranks, connecting rods and of the rocker, respectively, and J_{C1}, J_{C2}, and J_{C3}, are their mass moments of inertia calculated relative to their centers of mass; M^g_{red} is the reduced moment corresponding to the weight forces of the component elements of the mechanism of the conventional pumping units and M^f_{red} is the reduced moment corresponding to the force $\overline{F}$ acting at the end of the polished rod (fig. 7). M^g_{red} and M^f_{red} may be calculated with the following relations:

$$M^g_{red}=\frac{1}{\omega_1}\cdot(\overline{G}_1\cdot\overline{v}_{C1}+\overline{G}_2\cdot\overline{v}_{C2}+\overline{G}_3\cdot\overline{v}_{C3}+\overline{G}_{L1}\cdot\overline{v}_A+(\overline{G}_{L2}+\overline{G}_{tr})\cdot\overline{v}_B+\overline{G}_{CB}\cdot\overline{v}_{D'}+\overline{G}_{CG}\cdot\overline{v}_{A'}) \qquad (6)$$

$$M^f_{red}=\frac{\overline{F}\cdot\overline{v}_D}{\omega_1} \qquad (7)$$

The projections on the axes of the coordinate system *Oxy* (fig. 7) of the speed of any point *P* on the mechanism of the conventional pumping units may be calculated with the relations:

$$\begin{cases}(v_P)_x=\dfrac{dx_P}{dt}=\dfrac{dx_P}{d\varphi_1}\cdot\dfrac{d\varphi_1}{dt}=\omega_1\cdot\dfrac{dx_P}{d\varphi_1}\\[2ex](v_P)_y=\dfrac{dy_P}{dt}=\dfrac{dy_P}{d\varphi_1}\cdot\dfrac{d\varphi_1}{dt}=\omega_1\cdot\dfrac{dy_P}{d\varphi_1}\end{cases} \qquad (8)$$

where x_P and y_P are the coordinates of point *P* in the coordinate system *Oxy*.

Analogously may be calculated the angular speeds ω_2 and ω_3 of the connecting rods and of the rocker, respectively:

$$\omega_j=\dot{\varphi}_j=\frac{d\varphi_j}{d\varphi_1}\cdot\frac{d\varphi_1}{dt}=\omega_1\cdot\frac{d\varphi_j}{d\varphi_1};\quad j=2,3 \qquad (9)$$

where the angles φ_2 and φ_3 are represented in figure 7.

Relations (8) and (9) highlight the fact that J_{red}, M^g_{red}, and M^f_{red} do not depend on ω_1.

The manner of determining the positional parameters (the angles φ_{12} and φ_3 and the coordinates of the points intervening in relations 4, 6 and 7) depending on the dimensions of the component elements of the pumping unit and the crank angle φ_1 is presented in [8]. In [8] is also presented the manner of determining the crank angles φ_{1d} and φ_{1a} corresponding to the beginning of the upward and downward movements of the sucker rod column.

Results and discussions

The movement equation (1) has been transposed in a numerical form [10] by considering the successive positions of the pumping unit mechanism where have been performed the experimental records:

$$\frac{1}{2}\cdot\frac{J_{red,i+1}-J_{red,i}}{\varphi_{1,i+1}-\varphi_{1,i}}\cdot\omega_{1,i}^2+J_{red,i}\cdot\omega_{1,i}\frac{\omega_{1,i+1}-\omega_{1,i}}{\varphi_{1,i+1}-\varphi_{1,i}}=M_{red,i} \qquad (10)$$

where: $\varphi_{1,i}$ and $\varphi_{1,i+1}$ are the crank angles for two successive positions of the pumping unit mechanism; $\omega_{1,i}$, $J_{red,i}$ and $M_{red,i}$ are the values of the angular speed ω_1, of the reduced mass moment of inertia J_{red} and of the reduced moment M_{red}, respectively, when $\varphi_1=\varphi_{1,i}$.

The simulations have been performed with a computer program developed by the author using Maple programming environment [11]. It has been analyzed the case of a C-640D-305-120 pumping unit produced by *Lufkin* [13], whose component elements have the following dimensions: *OA* = 30 in. (0.762 m); *AB* = 133.5 in. (3.3909 m); *BC* = 111.09 in. (2.8217 m); *CD* = 155 in. (3.937 m). The coordinates of the point *C* (fig. 7) are [13]: x_C=(2.8194 m) and y_C= (3.5052 m). The values of the crank angles φ_{1d} and φ_{1a} are 88.976Ú and 266.982Ú, respectively. The values of the other parameters involved in calculations are: *OA'*=46 in. (1.1684 m); *OA"*=95 in. (2.413 m); *CD'*=140 in. (3.556 m); m_{L1}=88kg; m_{L2}=169kg; m_{tr}=*580 kg;* m_{CB}=840kg*;* m_{CG}=*4808kg;* q_1=722 kg/m; q_2=34 kg/m; q_3=300 kg/m, where q_1, q_2 and q_3 are the linear masses of the cranks, connecting rods and of the rocker, respectively.

In figure 8 is presented the variation of the reduced moment M^g_{red}. The curve *1* represents the variation obtained by applying the relation (6) and the curve *2* corresponds to the variation of the reduced moment M^g_{red} obtained using the relation (10) with the records concerning the variation of the force at the polished rod, of the motor moment at the crankshaft and of the angular speed of the cranks during the stroke 70. It can be noted the differences that appear around the values of the crank angles φ_{1d} and φ_{1a} (especially around the value of the crank angle φ_{1a}), where large variations of the recorded values of the angular speed of the cranks occur in short time intervals (fig. 2).

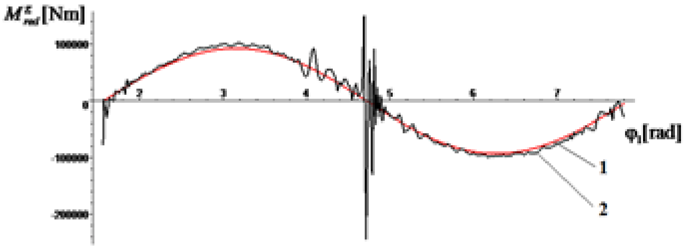

Fig. 8. The variation of the reduced moment M^g_{red}

By introducing the variation of the reduced moment M^g_{red} obtained by applying the relation (6) (the variation curve *1* in fig. 8) in the movement equation (10) has been calculated the variation of the motor moment M_m at the crankshaft during the strokes 71 and 72 (fig. 9 and fig. 10). The curves *1* represent the variation of the motor moment M_m obtained by simulation and the curves *2* correspond to the variation of its measured values for the two strokes.

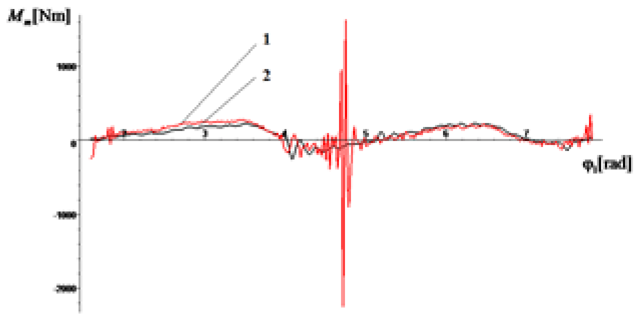

Fig. 9. The variation of the motor moment at the crankshaft during the stroke 71 (simulation -curve *1*; experimental - curve *2*)

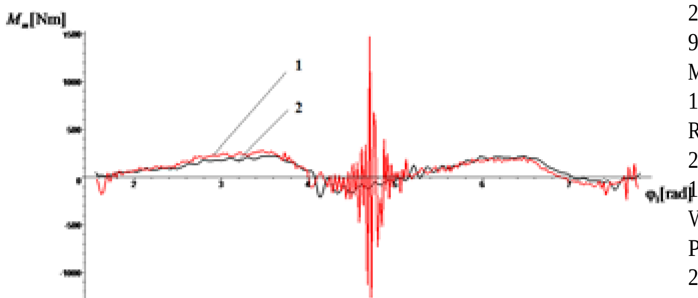

Fig. 10. The variation of the motor moment at the crankshaft during the stroke 72 (simulation - curve *1*; experimental - curve *2*)

Figures 9 and 10 highlight a good accordance between the simulation results and measured values of the motor moment M_m except the intervals of variation of the crank angle φ_1 around the values φ_{1d} and φ_{1a} (especially around the value of the crank angle φ_{1a}). This is due to the large variations of the recorded values of the angular speed of the cranks that occur in short time intervals (fig. 5 and fig. 6).

Conclusions

In this paper was analyzed the movement equation of the mechanism of the conventional pumping units. A series of simulation results have been presented in the case of the mechanism of a C-640D-305-120 pumping unit. It has been shown that the use of the numerical form of the movement equation presented in the paper leads to correct results for most of the operating cycle, differences occurring at the beginning of the upward and downward movements of the sucker rod column due to the large variations of the values of the angular speed of the cranks that occur in short time intervals.

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