Fatigue Life of the Process Equipment Made of Nonlinear Materials

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The fatigue life calculation for process equipment like chemical and petrochemical reactors, centrifuge, supercentrifuges etc. is important from the safety point of view. The fatigue life deduced in this paper depends on the effective dimension of the crack as well as on the mean stress, which represents a progress compared to the fatigue life value calculated based upon the stress intensity factor, where the mean stress is not taken into account. Using non-dimensional concepts, introduced by the critical energy principle, and by defining them in the case of fatigue loading of materials with cracks, a unitary theory was established for the fatigue life of nonlinear materials with cracks under cyclic loading.

Keywords: fatigue life, crack, deterioration, principle of critical energy, non-linear materials

Some of process equipment like chemical, petrochemical and nuclear reactors, shafts of mixing devices, of centrifuges, of supercentrifuges and disc-bowl centrifuges, as well as engines, ships, aircraft etc. are subject to fatigue.

The fatigue life depends on the stress amplitude and on material behaviour. Thus, the critical stress, σ_{cr} (e.g. fracture), diminishes over time due to material aging, material fatigue through cyclic stresses and creep. It is a stochastic parameter with values ranging inside a certain dispersion interval, $\sigma_{cr} \in [\sigma_{cr,\,min};\sigma_{cr,max}]$. The effective normal stress σ has a cyclic variation or a random variation between σ_{min} and σ_{max}. For a non-cracked material life time is a stochastic parameter ranging between $t_{l,min}$ and $t_{l,max}$, as it results from figure 1.

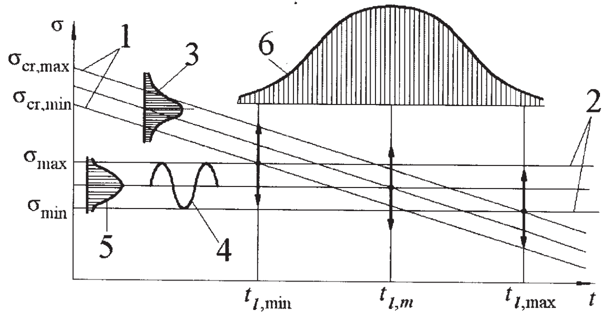

Fig. 1. Life time for a non-cracked material: 1 – critical stress dispersion interval; 2 – effective stress dispersion interval; 3 – σ_{cr} statistical scattering curve; 4 – stress cyclic variation, between σ_{min} and σ_{max}; 5 – effective stress statistical scattering curve, if it varies randomly between σ_{min} and σ_{max}; 6 – life time statistical scattering curve, t_l.

Philosophy of lifetime calculation, based on the critical energy principle

Relationships and a method for the calculus of fatigue life, based on the principle of critical energy and using the concept of participation of the action specific energy, will be deduced.

One considers materials with a non-linear behaviour at static loading, given by the power law relationships:

$$\sigma = M_\sigma \cdot \varepsilon^k \text{ and } \tau = M_\tau \cdot \gamma^k \qquad (1)$$

where:

σ, τ – normal stress and shear stress;

ε – strain;

γ – shear strain;

M_σ, M_τ, k, k_1 – material constants.

The calculation based on the concept of total participation can be done in two ways, which are:

a) – considering the random scatter of critical stress values and their dependence on the number N of loading cycles, $\sigma_{cr}(N)$ and $\tau_{cr}(N)$, case when the deterioration is comprised in the participation expression [1]:

$$P_\sigma(N) = \left(\frac{\sigma_a}{\sigma_{-1}(N)}\right)^{\frac{1}{k}+1} + \left(\frac{\sigma_m}{\sigma_u}\right)^{\frac{1}{k}+1} \cdot \delta_{\sigma_m} \text{ and}$$

$$P_\tau(N) = \left(\frac{\tau_a}{\tau_{-1}(N)}\right)^{\frac{1}{k_1}+1} + \left(\frac{\tau_m}{\tau_u}\right)^{\frac{1}{k_1}+1} \cdot \delta_{\tau_m}, \qquad (2)$$

where:

σ_a; τ_a is the amplitude of normal stress, and shear stress, and shear stress, respectively;

σ_m; τ_m – the mean normal stress, and mean shear stress;

$\sigma_{-1}(N)$; $\tau_{-1}(N)$ - fatigue strength after N loading cycles with normal and shear stresses;

σ_u; τ_u – ultimate stress or fracture strength at N = 1/4, at loading with normal and shear stresses, and

$$\delta_{\sigma_m} = \begin{cases} 1, \text{ when } \sigma_m > 0; \\ -1, \text{ when } \sigma_m < 0; \end{cases} \qquad \delta_{\tau_m} = \begin{cases} 1, \text{ when } \tau_m > 0; \\ -1, \text{ when } \tau_m < 0. \end{cases} \qquad (3)$$

Total participation as a sum of individual participations is also a function of N, and is a random value, too:

$$P_T(N) = P_\sigma(N) + P_\tau(N). \qquad (4)$$

Because the critical parameters diminish with the increase of number of loading cycles, meaning that $P_T(N)$ increases with the growth of N. Material deterioration was introduced through the use of critical parameters in $P_T(N)$ expression. Therefore, in this case $P_{cr} = P_{cr}(0)$ is constant and independent of N.

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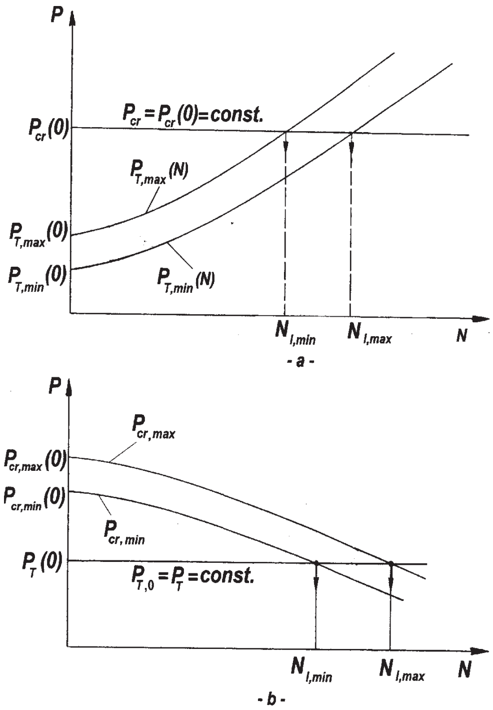

Fig. 2. The dependence on the number of cycles of the: total participation P_T and critical participation P_{cr}.

By solving the equation (fig. 2, *a*)

$$P_T(N)=P_{cr}(0), \qquad (5)$$

the fatigue life is obtained, N_l, taking values in the $N_{l,min}$; $N_{l,max}$ range, and thus being a random parameter as the strength values are $\sigma_{-1}(N)\in\left[\sigma_{-1,\min}(N);\sigma_{-1,\max}(N)\right]$ and $\tau_{-1}(N)\in\left[\tau_{-1,\min}(N);\tau_{-1,\max}(N)\right]$;

b) – without considering the dependence of critical parameters on N, but considering separately the deterioration produced by loading (including aging) in the expression of the critical participation. The total participation in this case is

$$P_T(0)=\bar{P}_\sigma(0)+\bar{P}_\tau(0). \qquad (6)$$

In this case P_T = $P_T(0)$ = constant, and the critical participation decreases with the increase of *N*, together with the increase of deterioration *D(N)* and it is calculated with the relationship,

$$P_{cr}(N)=P_{cr}(0)-D(N), \qquad (7)$$

which is a stochastic parameter. At t=0, before fatigue loading, $P_{cr}(0)\in\left[P_{cr,\min}(0);P_{cr,\max}(0)\right]$ is a probabilistic parameter, where $P_{cr,\max}(0)\le 1$ Solving the equation (fig. 2, *b*),

$$P_T(0)=P_{cr}(N), \qquad (8)$$

results the fatigue life expressed through the number of loading cycles until fracture, $N_l\in\left[N_{l,\min};N_{l,\max}\right]$.

Calculus of the fatigue life without considering cracks

In the case when the deterioration is introduced in the value of critical stress, for the calculus of fatigue life the principle of critical energy is used, along with the dependence σ_a(N) given by the Wohler curve. The Wohler curve is liniarized in the low cycle fatigue domain (I – fig. 3), that makes the calculus more safer from the engineering point of view.

The Wohler curve is split in three sectors to which correspond the three domains I, II, and III of figure 3 , which

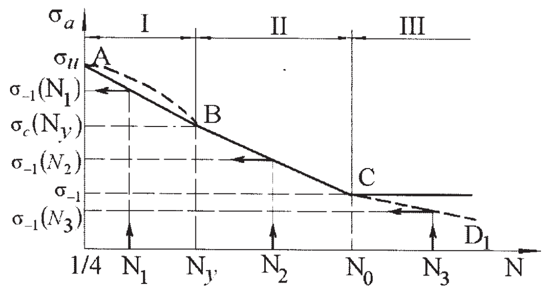

Fig. 3. The dependence σ_a(N) in the semi-logarythmic diagram (Wöhler curve), with the liniarization of it in the domain I

are described with relationships similar to Basquin relationship [2],

$$\sigma_a^m\cdot N=\text{constant}, \qquad (9)$$

where *m* is a material constant. One put $m=m_1$ for the domain I, $m=m_2$ for the domain II and $m=m_3$ for CD_1 in the domain III.

The fatigue life (N_l), is obtained with the use of relationships (2); (5) and (9) with $P_{cr}(0)=1$,

$$\left(\frac{\sigma_a}{\sigma_{-1}}\right)^{\alpha+1}\cdot\left(\frac{N_l}{N_0}\right)^{\frac{\alpha+1}{m}}+\left(\frac{\sigma_m}{\sigma_r}\right)^{\alpha+1}\cdot\delta_{\sigma_m}=1. \qquad (10)$$

where α=1/k. From the last relationship it is obtained the number of cycles until fracture. In the domain II, for example,

$$N_{l,2}=N_0\cdot\left(\frac{\sigma_{-1}}{\sigma_a}\right)^{m_2}\cdot\left(1-\left(\frac{\sigma_m}{\sigma_u}\right)^{\alpha+1}\cdot\delta_{\sigma_m}\right)^{\frac{m_2}{\alpha+1}}. \qquad (11)$$

For materials with fatigue limit (CD in fig. 3), the fatigue life is unlimited for σ_m = 0 and $\sigma_a\le\sigma_{-1}$. If $\sigma_m\ne 0$, then the following condition must be fulfilled,

$$\left(\frac{\sigma_a}{\sigma_{-1}}\right)^{\alpha+1}+\left(\frac{\sigma_m}{\sigma_u}\right)^{\alpha+1}\cdot\delta_{\sigma_m}=1, \qquad (12)$$

which, graphically represents the „diagram of constant lifetime" (fig. 4).

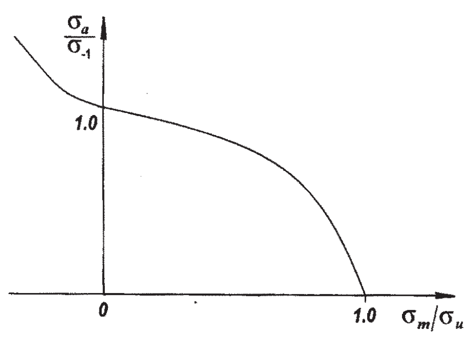

Fig. 4. The diagram of constant fatigue life (12)

In the case of fatigue loading by several blocks of normal stresses $(\sigma_{a,1};\sigma_{m,1})$; $(\sigma_{a,2};\sigma_{m,2})$ ... $(\sigma_{a,i};\sigma_{m,i})$ ...the fatigue life results from solving of the equation

$$P_T(\sigma)=1, \qquad (13)$$

where, e.g. for loadings with $\sigma_u\ge\sigma_{\max}\ge\sigma_y$ (domanin I in fig. 3),

$$\left(\sigma_{-1}(N)\right)_i=\sigma_y(N_y)\cdot\left(\frac{N_y}{N_{1,i}}\right)^{\frac{1}{m_1}} \qquad (14)$$

When the deterioration is introduced in the P_{cr} expression, the loading participation is constant over time ($P_T(N)$ = constant) and the deterioration produced by N loading cycles ($N < N_l$) is determined only by the stress amplitude σ_a. The deterioration relationship is

$$D(N)=\left(\frac{N}{N_l(\sigma_a)}\right)^{\frac{\alpha+1}{m}}, \qquad (15)$$

where $N_l(\sigma_a)$ is the number of cycles to failure under stress amplitude σ_a.

Through particularizations, e.g. for the domain II of Wöhler curve, one obtains:

$$D(N)=\left(\frac{N}{N_{l,2}(\sigma_a)}\right)^{\frac{\alpha+1}{m_2}} \qquad (16)$$

The total deterioration, at loading with several blocks of stresses $(\sigma_{a,i};\sigma_{m,i})$, is calculated with the relationship

$$D_T(N)=\sum D_i(N). \qquad (17)$$

Calculus of the fatigue life considering the cracks

The points of concentration for stresses, sharp edges or material defects, represent points for cracks' initiation. Once the crack started, it advances stable and uniformly with a finite value, at each loading cycle. When reaching a certain value of crack length, its extension becomes unstable, followed by rupture. Deterioration at a certain time, determined by a crack that reached the semi-length $a(t)$, according to the principle of critical energy has the expression [3],

$$D(a)=\left(\frac{a(N)}{a_{cr}}\right)^{\frac{\alpha+1}{2}} \qquad (18)$$

where $a(N)$ is the crack characteristic dimension after N loading cycles and a_{cr} – the crack critical characteristic dimension under loading conditions.

The cyclic loading participation with normal stresses $(\sigma_{a,1};\sigma_{m,1})$, corresponding to a number N of effective cycles is

$$P_\sigma(N)=\left(\frac{N}{N_l(\sigma_a)}\right)^{\frac{\alpha+1}{m}}+\left(\frac{\sigma_m}{\sigma_u}\right)^{\alpha+1}\cdot\delta_{\sigma_m}. \qquad (19)$$

Equaling the effective participation with the critical one,

$$P_\sigma(N)=1-D(a),$$

results the fatigue life of the cracked material, under fatigue loading with the stress amplitude $\sigma_{a,1}$ and the mean stress $\sigma_{m,1}$.

$$N_l=N_l(\sigma_{a,1})\cdot\left[1-\left(\frac{\sigma_{m,1}}{\sigma_u}\right)^{\alpha+1}\cdot\delta_{\sigma_m}-\left(\frac{a(N)}{a_{cr}}\right)^{\frac{\alpha+1}{2}}\right]^{\frac{m}{\alpha+1}}, \qquad (20)$$

where m is replaced with m_1; m_2 or m_3 depending of the domain I, II or III of the Wöhler curve where is used and $N_l(\sigma_{a,1})$ is the number of alternating symmetrical loading cycles (σ_m=0) until fracture, of the same material but without cracks (a=0).

In the case of several blocks loading, from the previous relationship the following general relationship can be obtained:

$$\sum_i\left(\frac{N_i}{N_{l,i}(\sigma_{a,i})}\right)^{\frac{\alpha+1}{m}}=C, \qquad (21)$$

where:

$$C=\left[1-\left(\frac{\sigma_m}{\sigma_u}\right)_f^{\alpha+1}\cdot\delta_{\sigma_{m,i}}-\left(\frac{a}{a_{cr}}\right)^{\frac{\alpha+1}{2}}\right], \qquad (22)$$

a is the crack with the most dangerous evolution and $(\sigma_m/\sigma_u)_f$ is the value of this ratio for the final (last) block of load.

For the alternating symmetrical loading (σ_m=0) of a material without cracks (a=0), C=1, so that relationship (21) transforms into

$$\sum_i\left(\frac{N_i}{N_i(\sigma_{a,i})}\right)^{\frac{\alpha+1}{m}}=1. \qquad (23)$$

If the exponent $\frac{\alpha+1}{m}=1$ relationship (23) becomes the Palmgren –Miner low [4; 5], and for $\frac{\alpha+1}{m}=p$ one obtains the Marko-Starkey relationship [6]. The possible values for the exponent of relationship (23) are obtained based on values k and m. Thus, for steels: k=0.16...1.0; m=3...5; α=1/k=1...6.25 - forstatic loading; α=1/2k=0.5...3.125 – for rapid loading; α=0 – for shock loading. Therefore, $\frac{\alpha+1}{m}\in[0.2;2.4]$.

The theoretical values for the exponent of relationships (21) and (23) are comprised between the possible values experimentally determined (0.6 and 1.0).

Experimentally it was observed that the fracture occurred at C values both sub- and supra-unitary. Miner showed that fracture occurred at C values between 0.61 and 1.49 [5]. Depending on the number of loading cycles, of the initially applied stress (high or low) a dispersion of the C values between 0.1 and 10 was obtained; it was reported in [7]. In the calculus standards (e.g. BS 5500; EN – 13445-3) the Palmgren-Miner relationship is used in the following form:

$$\sum_i\left(\frac{N}{N_l}\right)_i=C_1, \qquad (24)$$

where C_1, for pressure vessels with welding defects is recommended (EN-13445-3):

$$C_1=\begin{cases}0.8\text{ - for }N_{ech}\in[500;\ 1{,}000);\\0.5\text{ - for }N_{ech}\in[1{,}000;\ 10{,}000];\\0.3\text{ - for }N_{ech}>10{,}000,\end{cases} \qquad (25)$$

where N_{ech} is the number of equivalent loading cycles, calculated according to this standard. A lot of experimental tests showed that C_1=1/2 ÷ 2/3, reason for which it was adopted at a certain moment C_1=0.5. Relationship (PD 5500/2003) $C_1=0.6\cdot\left(\frac{22}{s}\right)^{0,75}$, where s is the highest thickness value of the wall under loading (s ≥ 22 mm), but at least 22 mm. Here, always C_1≤ 0.6. Afterwards, it was recommended C=1 according to ASME Code Section VIII, Division 3. All these recommendations are based on experiments and have not an adequate theoretical base.

In conclusion, the general relationship (21) for the calculus of the fatigue lifeat cyclic loading, comprises as particular cases the Palmgren - Miner relationship, as well as the Marko-Starkey relationship, both experimentally obtained.

Conclusions

Analyzing the causes that determine the fatigue life of materials under fatigue loading conditions resulted that always we do encounter a distribution of fatigue lifes and not a unique value of it. For the calculus of fatigue life, in this paper, were proposed relationships for materials without cracks as well as for materials with cracks that propagate during loading. The fatigue life calculated in this way depends on the effective dimension of the crack as well as on the mean stress, which represents a progress compared to the fatigue life value calculated based upon the stress intensity factor, where the mean stress is not taken into account. Using non-dimensional concepts, introduced by the critical energy principle, and by defining them in the case of fatigue loading of materials with cracks, a unitary theory was established for the fatigue life of materials with cracks under cyclic loading.

Nomenclature

$D(a)$; $D(N)$	deterioration or damage due to crack and due to number of cycles respectively;
M_σ; M_τ	material constants;
N; N_l	number of loading cycles, fatigue life, respectively;
$P_\sigma(N)$; $P_\tau(N)$	participation of specific energy due to effective number of cycles (N) in the case of normal stress (σ) and shear stress (τ), respectively;
$P_T(N)$	Total participation of specific energy after N cycles of loading;
P_{cr}	critical value of specific energy participation;
$R=\sigma_{min}/\sigma_{max}$	stress ratio;
$2a$; $2a_{cr}$	total crack length, total critical crack length, respectively;
k; k_1	exponents in the laws of behaviour;
m	exponent in Basquin's relationship;
m_1; m_2; m_3	the values of m in the domains I, II, III (Fig. 5), respectively;
t; t_l	time, life time, respectively;
ε	strain;
γ	shear strain;
σ; σ_{cr}	applied normal stress, critical normal stress, respectively;
σ_a; σ_m	amplitude of normal stress, mean normal stress, respectively;
σ_u	ultimate normal stress;
σ_y	normal yield stress;
σ_{-1}	normal stress fatigue limit;
τ; τ_{cr}	shear stress, critical shear stress, respectively;
τ_a; τ_m	amplitude of shear stress, mean shear stress, respectively;
τ_u	ultimate shear stress;
τ_y	yield shear stress;
τ_{-1}	shear stress fatigue limit

Acknowledgements: The author wish to acknowledge the CNCSIS (National Council of Scientific Research in Higher Education) in Romania for their financial support for this work.

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Manuscript received: 13 10. 2011